

NMCC 2017



Road selection in a grid

Report

Sweden

Täby Friskola year 8

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Introduction

This year's task is a survey on the number of different routes between two positions in a grid. The ant, which is moving in the grid, must follow a certain number of rules and therefore has a limited number of choices.

Our interpretation of the rules says that the ant may move on any of the vertical lines between the two positions S and M, including the vertical lines where S and M are placed. However, the ant is not allowed to return to a position previously visited and it may not move upwards in the grid.

Our report will describe two methods that show us how we solved the problem.

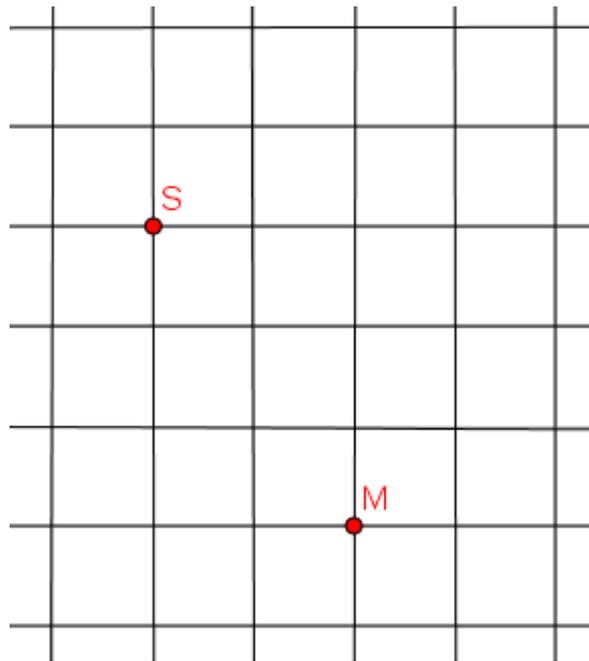


Figure 1

Method 1: The Combinatorial Solution

When we received the message that we had qualified to the semi-finals we all were overwhelmed and cheered out loud. We were all eager to see the given task and to start working with it. The task immediately piqued our interest, because it was something new for us. It's also something we can use in our daily life. For example we have a game in school every year that is called "killergame". The goal is to "kill" other players and avoid getting killed. To win the game, you need to make a strategy to avoid other players, therefore you need to know every possible routes.

We split the class into groups with four students in each and every group was provided with an A3 paper on which we wrote down our ideas on the matter. We compared our results and agreed on the number of different choices the ant has.

The head of the Swedish Sigma-8 competition, Bengt Åhlander, came to our class and we gave him a warm welcome as he helped us to interpret the task. He held a short lecture in order for us to get a better view on the problem. In the beginning, the class disagreed on whether the ant was allowed to travel upwards to a line that it already had passed or not. But after Bengt's lecture, we all agreed that the ant is not allowed to do so. Then we started to work with the problem in a mathematically correct way.

The first solution rests on the following thoughts:

The positions S and M are located on two different horizontal lines and between them are two more lines that are horizontal. According to the rule that the ant is not allowed to travel on a horizontal line more than once, the ant cannot travel above the position S or below the position M. The reason is that it will have to cross a horizontal line that it has already stepped on, in order to reach the final destination (the target node): position M. The conclusion is that the horizontal lines 1, 2 and 7, marked with red in *Figure 2*, are out of the question.

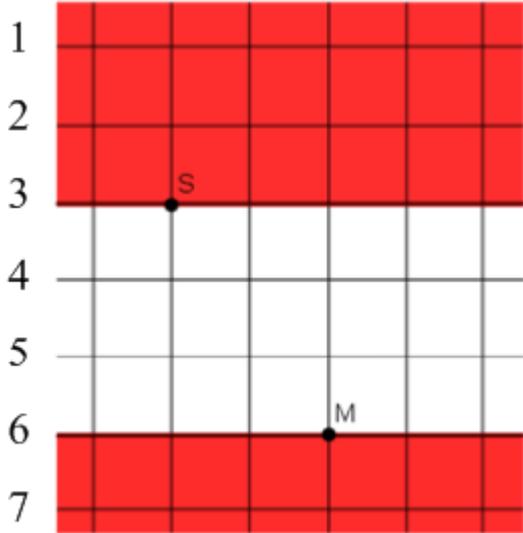


Figure 2

There are six different alternatives for the ant to get from its starting position, S, to the horizontal line 4, because there are six vertical lines in the grid. If the ant travels down a vertical line to get to the end point, it has to go straight to it, according to the previously presented rule. In the next sequence, the ant will have another six options to reach line 5 from line 4. This will also be the case when moving from line 5 to 6. From line 6 there is only one way to reach position M. The solution given is:

$$6 \cdot 6 \cdot 6 \cdot 1 = 6^3 = 216$$

With this knowledge we constructed the formula:

$$N = V^{H-1}$$

N is the number of possible routes.

V is the number of vertical lines.

H is the number of horizontal lines that the ant can travel on.

An example on how we can use the formula described above:

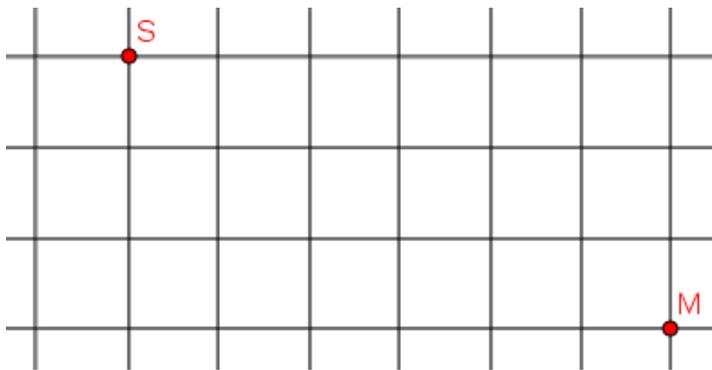


Figure 3

V: 8

H: 4

$$N = 8^{4-1} = 8^3 = 512$$

The formula is able to calculate the number of road selections in an infinite grid size, and for any given start and end position. We found that this method works well, because of the simplicity to adapt it to any grid size.

Method 2: The Programming Solution

While some of the students in our class developed the first method, the rest of the class tried to find alternative solutions. One idea was to count the number of possible routes using the number of turns that the ant takes in the grid. We stopped working with that method since we could not find any correlation between the number of turns and the number of routes. The second method that we applied was to count the number of possible routes using a certain distance between S and M. Initially we aimed at drawing all possible routes. After having marked the shortest routes, which are 5 lengths, and the longest, which is 19 lengths, however we quickly realized that it would become too complicated. It would for instance take a very long time just to draw the 62 different routes that have the length 9.

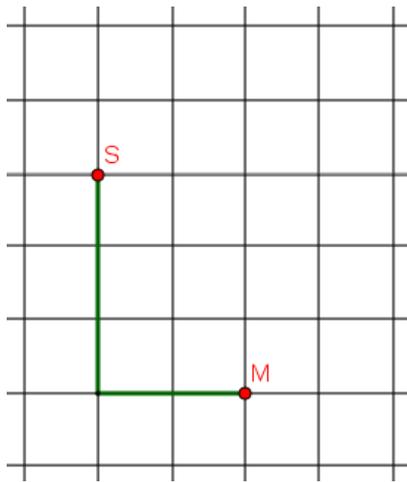


Figure 4

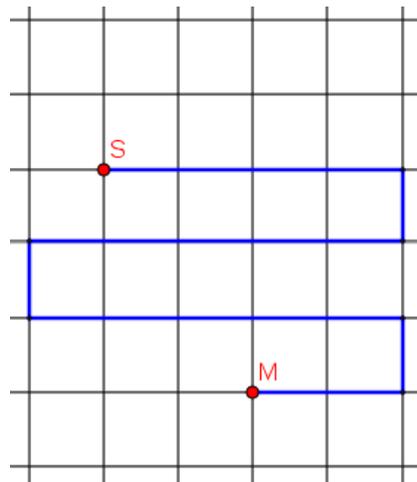


Figure 5

When none of our ideas in the beginning worked out, it had us upset and frustrated. It seemed impossible to find a second solution. But, Noel Pellkvist, one of the students in our class with programming skills, came up with the idea to solve the problem using programming.

After a lecture by Noel on how the Microsoft Visual Studio tool works, we began to create a program together, which can calculate the number of possible routes that the ant can take in a grid and which fulfils the given criteria. When choosing this tool, we got some assistance from Lisa Vällfors, one of Anna's former students, who today studies IT at KTH.

During the programming work, we found out that the code was incorrect, because we had accidentally inverted the y-coordinates, which resulted in everything in the grid being upside down. With the help of Noel's dad, Johan Pellkvist, we realized that we had to fix the y-coordinates so the program could run correctly.

We used the program to determine the number of possible routes for a given distance. We start with a grid of 7 x 6 lines, just as the grid in the original problem. According to the rules of the problem, the ant is only allowed to walk on the horizontal lines between the nodes S and M and the horizontal and vertical lines in between. This means that the grid that the ant is allowed to walk is 4 x 6 lines.

To create the program, which determines the number of possible routes in a grid, we used the tool: Microsoft Visual Studio. All programs are made up of a code that prescribes how it works and what has to be done. See *Figure 6* to get some insight on what the code for our programming solution looks like. The program we used creates different variables for the horizontal lines, where the ant must make a choice (on the line which hosts the target node (M) the ant does not have to make any choice but can walk straight to the target node, therefore that line needs no variable).

```
118     {
119         pathsa.Clear();
120         for(int V1 = 0; V1 < vågräta; V1++)
121         {
122             for (int V2 = 0; V2 < vågräta; V2++)
123             {
124                 for (int V3 = 0; V3 < vågräta; V3++)
125                 {
```

Figure 6

The first variable, which we call V1, decides at which node the ant turns downwards from the first horizontal line (*Figure 7*). After that the second variable, which we call V2, decides the node at the second horizontal where the ant makes a downturn. Our third variable, V3, does the same thing but on the third horizontal line. Just as we described earlier the last horizontal line does not need any variable as the ant neither makes a downturn nor has to choose any new node. The only thing the ant needs to do on the last horizontal line is to walk to the target node. Using this method, we can solve all problems with the ant in different grids. In case the number of possible routes would exceed to 3,7 million, our computer will become over-loaded and stop working.

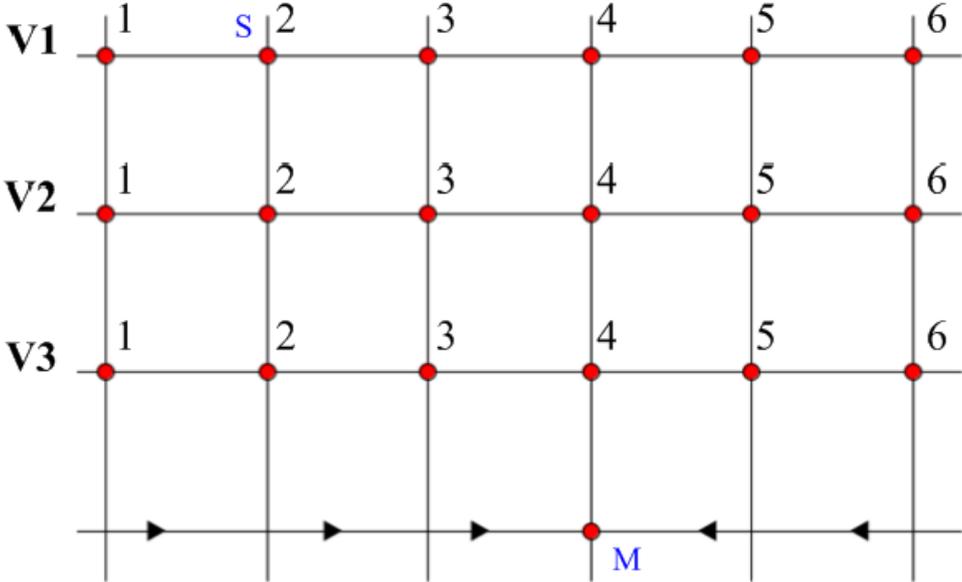


Figure 7

Using our program we can determine the number of possible routes with a certain distance in different grids. The results from *Figure 8* are we showing in *Table 1*.

Length of the shortest route: 5 (*Figure 4*)
 Length of the longest route: 19 (*Figure 5*)

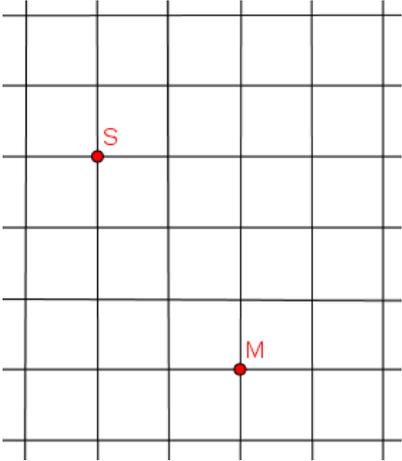


Figure 8

The lengths of different routes:

Lengths	5	7	9	11	13	15	17	19
Number of routes	10	40	62	47	32	20	4	1

Table 1

If we add the number of routes in the table we reach the sum 216, which is the same as we reached in the first method.

The total number of routes:

$$10+40+62+47+32+20+4+1=6^3= 216$$

New Start Nodes And End Nodes

After having reached the answer in the original grid, we chose two different starting and end nodes in the same grid. Then we determined the number of possible routes using our two methods. We got the following results:

$$N=V^{H-1}$$

N is the number of possible routes

V is the number of vertical lines

H is the number of horizontal lines that the ant can walk

Example 1

$$V=6, H=6$$

$$N=6^{6-1}=6^5=7776 \text{ routes}$$

Length of the shortest route: 9 (*Figure 10*)

Length of the longest route: 31 (*Figure 11*)

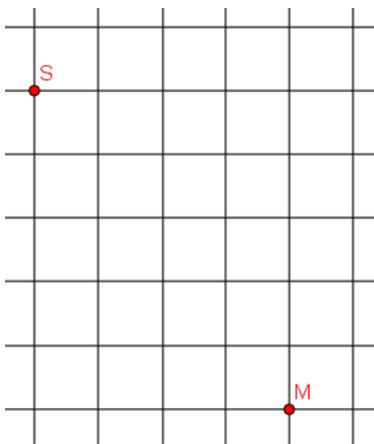


Figure 9

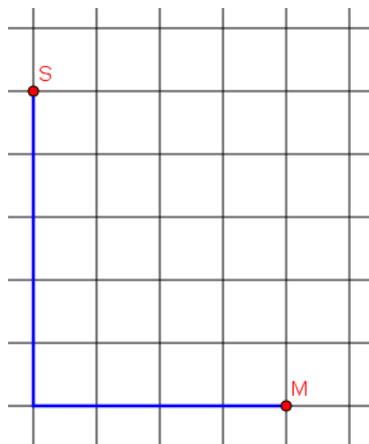


Figure 10

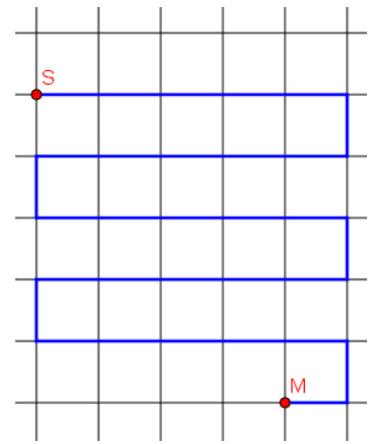


Figure 11

The lengths of different routes:

Lengths	9	11	13	15	17	19	21	23	25	27	29	31
Number of routes	126	546	975	1401	1653	1527	771	415	230	102	29	1

Table 2

The total number of routes:

$$126+546+975+1401+1653+1527+771+415+230+102+29+1=6^5=7\,776$$

Example 2

$$V=6, H=3$$

$$N=6^{3-1}=6^2=36 \text{ routes}$$

Length of the shortest route: 3

Length of the longest route: 13

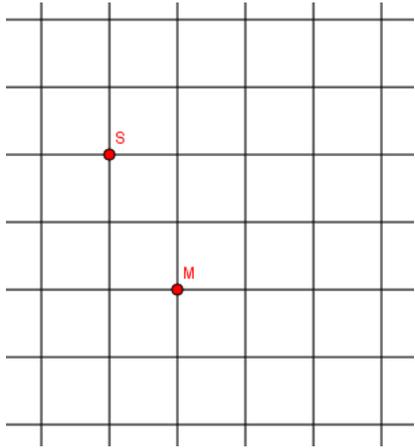


Figure 12

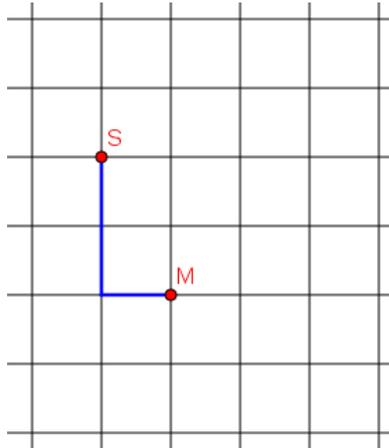


Figure 13

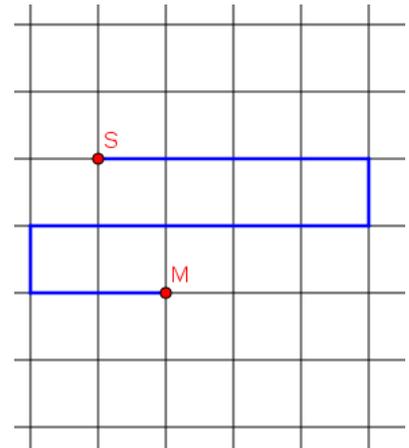


Figure 14

The lengths of different routes:

Lengths	3	5	7	9	11	13
Number of routes	3	5	7	9	11	1

Table 3

The total number of routes:

$$3+5+7+9+11+1=6^2=36$$

We expand the grid

We expand the grid and place the start and target nodes at new places. We show this in three different examples.

Example 1

$8^5 = 32\,768$ routes

Length of the shortest route: 11

Length of the longest route: 41

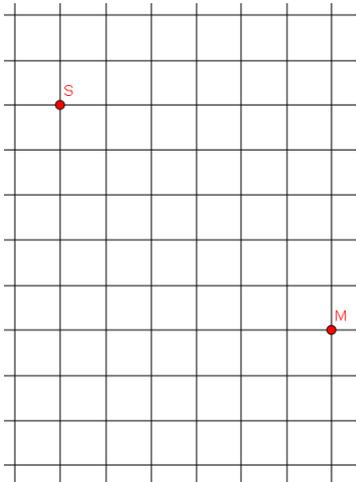


Figure 15

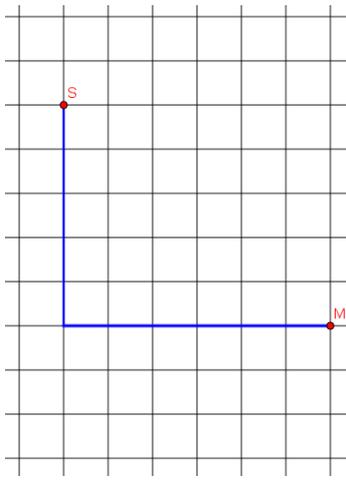


Figure 16

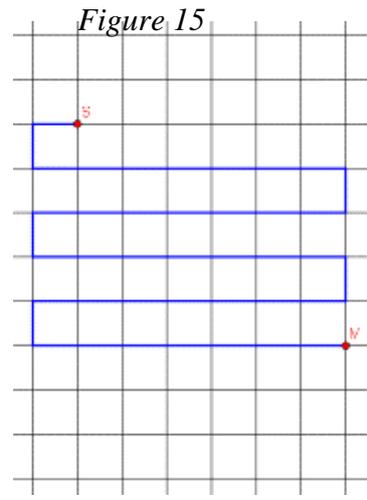


Figure 17

The lengths of different routes:

Lengths	11	13	15	17	19	21	23	25	27
Number of routes	462	1485	2409	3473	4439	5091	5165	4349	2283
	29	31	33	35	37	39	41		
	1484	1001	615	330	142	39	1		

Table 4

The total number of routes is:

$$462 + 1485 + 2409 + 3473 + 4439 + 5091 + 5165 + 4349 + 2283 + 1484 + 1001 + 615 + 330 + 142 + 39 + 1 = 32\,768.$$

Example 2

$6^0=1$ routes

Length of the shortest route: 6

Length of the shortest route: 6

Lengths	6
Number of routes	1

Table 5

With this start and target node, there is only one route.

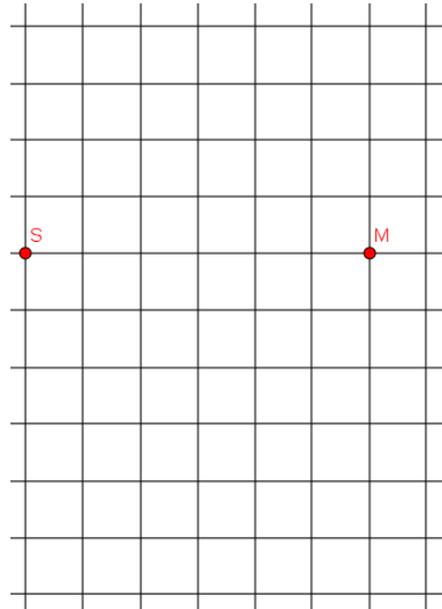


Figure 18

Conclusion

Our program can quickly determine the number of possible routes an ant can take to traverse any grid between two endpoints. With our computer we were able to calculate up to approximately 3,7 million route choices.

Compared to Method 1, Method 2 requires a computer with high performance and a lot of experience in programming. Without Noels knowledge, Method 2 would not have been an option for us. A normal calculator can be used with Method 1 to process much larger numbers, than a normal computer using our program. Method 1 can be used for very large numbers and grids, while Method 2 is more efficient, once the program has been implemented. The combinatorial solution can only give the total number of route choices, while our program can give us information regarding all route choices and their lengths.

Something we have discovered while working with this task is that every grid, where the start and the end points are not on the same vertical line, have at least two shortest lengths and only one longest length. However, something even more interesting is when the start and the end points are on the same vertical line, there will be two different routes with the longest length (*Figure 21*), but only one route with the shortest length (*Figure 20*). With that comparison, can we infer that the second grid (*Figure 19*) is the only exception where there is only one shortest length and two longest lengths.

Summary

Our group has been working with a grid where an ant travels between start and end points, and at the same time follows certain rules. Our goal was to answer the questions in the assignment, but also to learn something new, which we have achieved. We were able to solve the problem in multiple ways and chose to present two solutions in our report: a combinatorial and a programming solution.

The work has not only given us new knowledge in the realm of mathematics, it has also strengthened the cohesion of our class and taught us the benefits of working as a team. Among other things, we have learned to report and explain mathematics at a more advanced level and have also received experience in programming.

Our teacher Anna has had a lot to do since she was awarded the title “Sweden’s best math teacher.” Anna has not been able to be constantly present to give us practical advice and this forced us to support and be reliant on each other. We have benefited from the opportunity to work and think independently in our group. Erland Arctadius was available to advise us on the rare occasion when we needed extra assistance.

The ups we experienced with breakthroughs were combined with difficulties that tested our motivation. It really has been an emotional roller coaster. The exhaustive and constant testing of various methods required persistence and tested our determination.

All of the tribulations were worth the effort and we exaltedly reached our goal!

Sources

Books

- Hans Wallin, Rolf Axelsson, Gunilla Jakobsson, Lars Jakobson and Klas Nilson (2002) *Diskret MATEMATIK för gymnasiet*. Liber.
- Attila Szabo, Niclas Larson, Gunilla Viklund, Daniel Dufåker and Mikael Marklund (2013) *Matematik origo 5*. Sanoma utbildning.
- Bengt Ulin (1993) *Liten guide för matematiska problemlösare*. Liber.
- Christer Kiselman and Lars Mouwitz (2008) *Matematiktermer för skolan*. NCM.
- Susanne Gennow, Ing-Mari Gustafsson and Bo Silborn (2011) *Matematik för gymnasiet exponent*. Gleerups Utbildning AB.

Websites

- <https://www.visualstudio.com/downloads/>

People

- Erland Arctaadius
- Lisa Vällfors
- Bengt Åhlander
- Johan Pellkvist

Photo

- Malin Eklöv