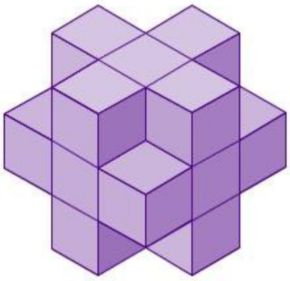
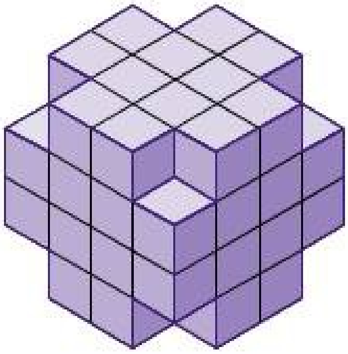


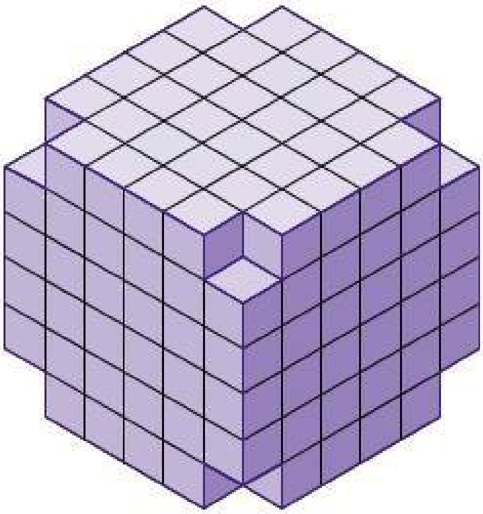


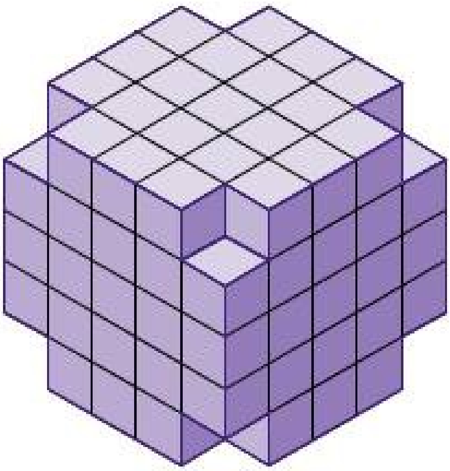
Process log

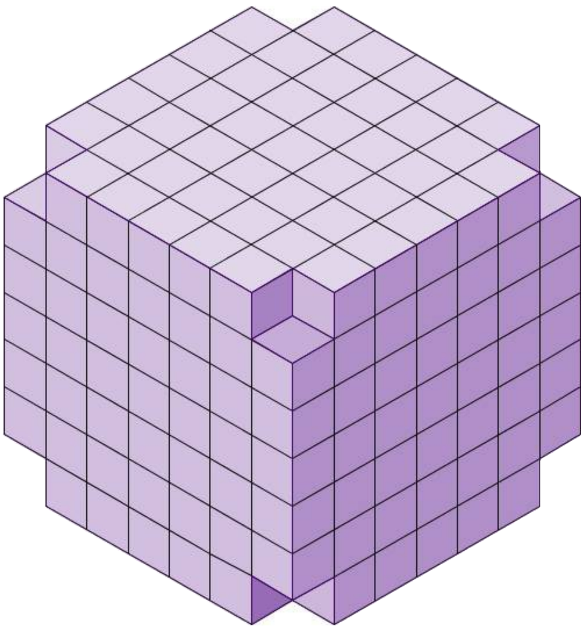
Nordic Math Class Competition 2016

Denmark

Kongevejens Skole 8.y







Wednesday, March 30th:

Lesson 1 from 12:45 to 13:30 and lesson 2 from 13:30 to 14:15.

The teachers introduced the class to the assignment. They recommended that in the first two lessons, we should work separately or in groups of two.
We started by reading the assignment, part A, and then started to find different ways to solve task a) and b).

For most of us, the assignment called for a new way of solving mathematical problems. Most of us had not tried this kind of task before, and it was new to us that the whole class would have to collaborate to solve the assignment.

After a while, everyone was working hard on finding solutions. Some decided that they would draw the figures, and after 20 minutes, figures 1-5 had been drawn.

Monday, April 4th:

Lesson 3 from 8:00 to 8:45 and lesson 4 from 8:45 to 9:30.

Today the class worked in groups of four or five. We compared our solutions from lessons 1 and 2, and came up with new solutions. Some of the groups discovered new ways of solving the assignments. Some of our solutions were considered useless.
The groups decided which solutions they liked the best, in most groups it was the same.

At the end of lesson 3, we had six different usable solutions.
All groups started to work with the rest of the assignment – c), d) and e), and some started to compare their answers with other groups.

Wednesday, April 6th:

Lesson 5 from 12:45 to 13:30 and lesson 6 from 13:30 to 14:15.

Today the class' goal was that all groups would be done with their wording of the assignment solution. In lesson 5, most groups still worked with the assignment's parts c), d) and e).

In lesson 6 all the groups were working on their wording. All the wordings were done at the end of the lesson. We collected all the wordings in a plastic folder. It was up to a smaller group to include the wordings into the report.

Monday, April 11th:
Lesson 7 from 8:00 to 8:45 and lesson 8 from 8:45 to 9:30

Today the hard work began. Each group had written a wording *and* a process log. In the end, we were only supposed to have one report and one process log.

We decided who were going to represent the class. We chose another group of four; they were responsible for the writing of the report and process log.

Wednesday, April 13th:
Lesson 9 from 12:45 to 13:30 and lesson 10 from 13:30 to 14:15.

Today was mostly about centicubes. The report was almost done, but we felt pressure from the time. Our submission date was tomorrow. It was very difficult to turn five different wordings, made by 24 people, into a single report.

Thursday, April 14th:
Lesson 11 from 9:50 to 10:35.

Submission date!
The last day of hard work. The whole class felt the pressure. It was a relief to all, when we were done. Finally we thought... for a while.

From now on the log mostly concerns the four representatives and they are also the only ones continuing working with the report.

Monday, May 2nd:

Lesson 2.1 from 8:00 to 8:45 and lesson 2.2 from 8:45 to 9:30.

The class was informed about the international part of the assignment. The report and the process log should be written in English. The four representatives were chosen for translating the report and process log.

The hard work of translating from Danish to English started. We were told that our Danish report lacked illustrations, so most of the lessons went with drawing illustrations of the figures in GeoGebra.

Tuesday, May 3rd:

Lesson 2.3 from 12:45 to 13:30 and lesson 2.4 from 13:30 to 14:15.

Today we worked with the translation of the report. Most of the time, we sat with each our dictionary and tried to work out what different words meant.

Wednesday, May 4th:

Lesson 2.5 from 12:45 to 13:30 and lesson 2.6 from 13:30 to 14:15.

Today we were done with the illustrations for the report. We started to translate our process log too.

And then...

We all went on a school camp,

Wednesday, May 18th:

Lesson 2.7 from 12:45 to 13:30 and lesson 2.8 from 13:30 to 14:15.

Once again, we worked on our translation. Two with the report and two with the process log. We are almost done with them. Again, we felt the pressure from time. Translating math from Danish to English was more difficult than first thought.

Thursday, May 19th:

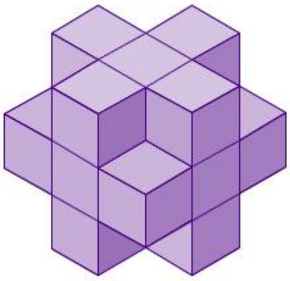
Lesson 2.9 from 12:00 to 12:45, and some time after school.

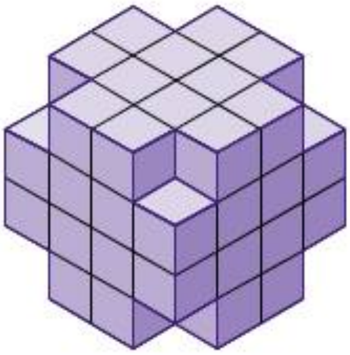
Today we discussed our translation with our English teacher. And finally, finally we were done.

The whole class agreed that the process, though difficult, has been very exciting and educational.

Kongevejens Skole 8.y:

Anders, Andreas, Eva, Frederik, Gustav, Ida, Jens, Julius, Kathrine, Kristian, Laura, Ludvig, Magnus, Maise, Marie, Marius, Olga, Oliver, Parsa, Sara, Simone, Sofie, Victor, Waugh.



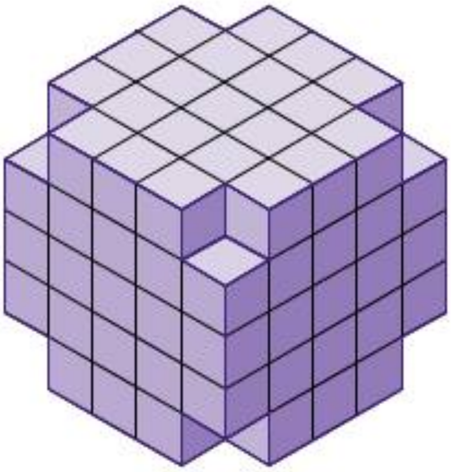


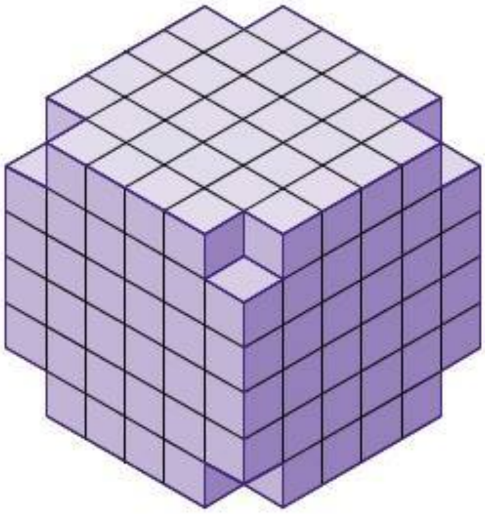
3D-figure

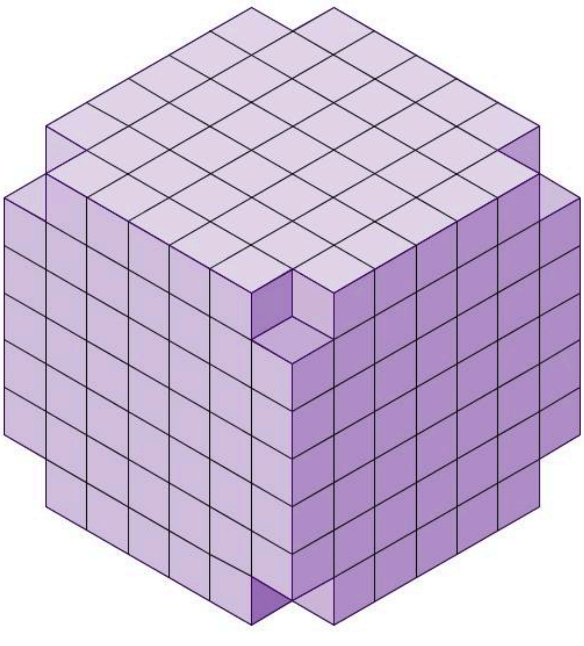
Nordic Math Class Competition 2016

Denmark

Kongevejens Skole 8.y







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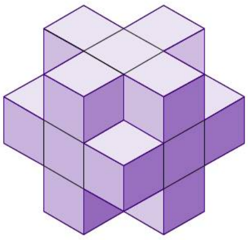
Introduction:

We chose to solve the first part of the assignment by making equations and creating the figures in as many ways as possible. We chose to abbreviate *figure number* as ***n.*** We also chose to abbreviate *number of cubes in a figure* as ***V.*** It makes the process more clear. We were divided into groups, so a couple of groups draw and built the figures and the other groups tried to create equations.

a)

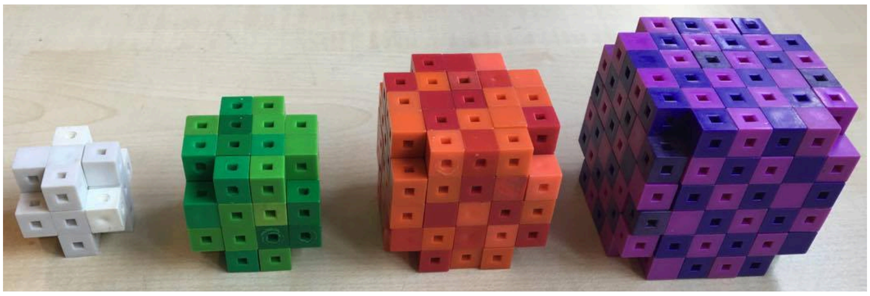
Connection between figure number and number of cubes:



We chose to draw the figures, not as a solution, but because we would be able to discuss our solutions while looking at the drawings. We think that it is a lot easier to discuss solutions, when we have a drawing as a base for our discussion.

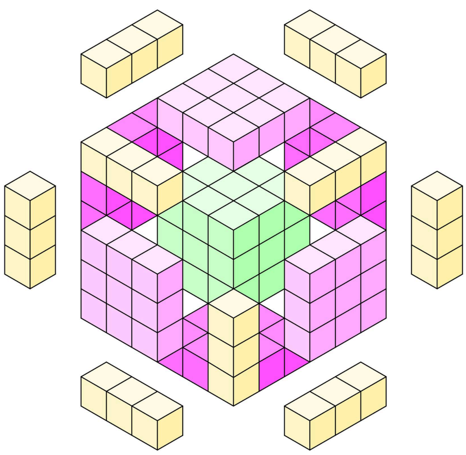
Solution a)

Construction of figures in centicubes.



By constructing the figures in centicubes, it's easy to count how many cubes we used for each figure. We can, among others, use that to find the increase in the number of cubes. We can also divide the figures into parts that make it possible for us to create equations for the figure's volume.

Solution b)

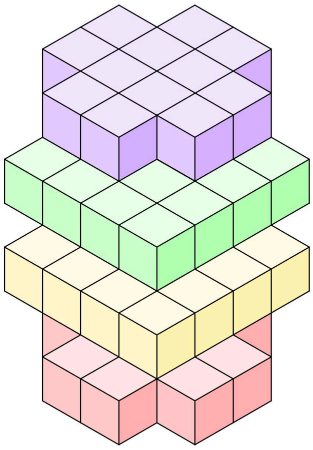


Division of the figures into minor parts.

This can be a continuation of building the figures, or it can be something you calculate. Every figure has a core that consists of n^3 cubes (green). Around the core, there are 6 surfaces with each n^2 cubes (pink). Finally, we have 12 "bars", which each consists of n cubes (yellow). That gives us

$\mathbf{V = n^3 + 6n^2 + 12n}$ cubes in each figure.

Solution c)



Division of the figures in layers.

Instead of dividing the figures in 19 parts, we can divide them in layers. By building, drawing or simply thinking, you can see that every figure contains:

- a top- and bottom layer with each $n^2 + 4n$ cubes
- n layers with each $(n + 2)^2$ cubes

Therefore $\mathbf{V = 2(n^2 + 4n) + n(n + 2)^2}$.

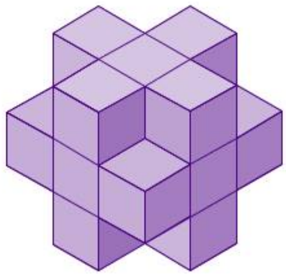
Reduced:

$2n^2 + 8n + n(n^2 + 4n + 4)$

$2n^2 + 8n + n^3 + 4n^2 + 4n$

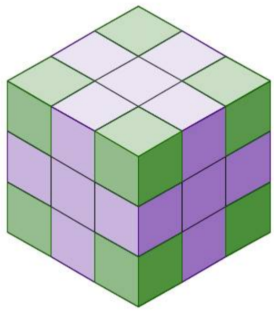
$\mathbf{V = n^3 + 6n^2 + 12n}$

Solution d)



Thinking of the figures as square blocks without the corner cubes.

If you picture the figures as square cubes, you can see that every side has a length of n + 2. To find the area of the whole cube, you use $(n + 2)^3$. All the figures lack eight corner cubes and therefore $\mathbf{V = (n + 2)^3 – 8}$.



Reduced:

$(n + 2) * (n + 2) * (n + 2) – 8$

$(n^2 + 4n + 4) * (n + 2) – 8$

$n^3 + 4n + 4n^2 + 2n^2 + 8 + 8n – 8$

$\mathbf{V = n^3 + 6n^2 + 12n}$

Solution e)

The cube divided in inside and outside.

We thought that you could divide the cube in a: cubes without visible surfaces and b: cubes with visible surfaces. The cubes without visible surfaces, the core, is n^3. If you calculate the number of cubes for one side, the center is n^2. The outer parts are n*4. That means a surface is $n^2 + n * 4$. We have 6 surfaces, and that gives us $(n^2 + n * 4) * 6$. The edges of every side (n * 4) overlap each other so we have to subtract the surplus edges, which gives us $(n^2 + n * 4) * 6$. Then we add the cubes with no visible surfaces, the core:

$\mathbf{V = (n^2 + n * 4) * 6 – 12n + n^3}$

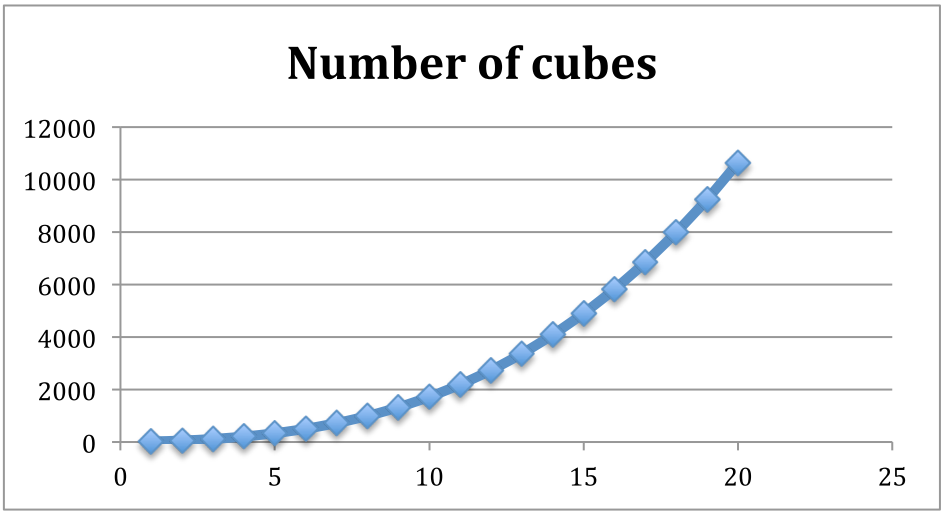
Reduced:

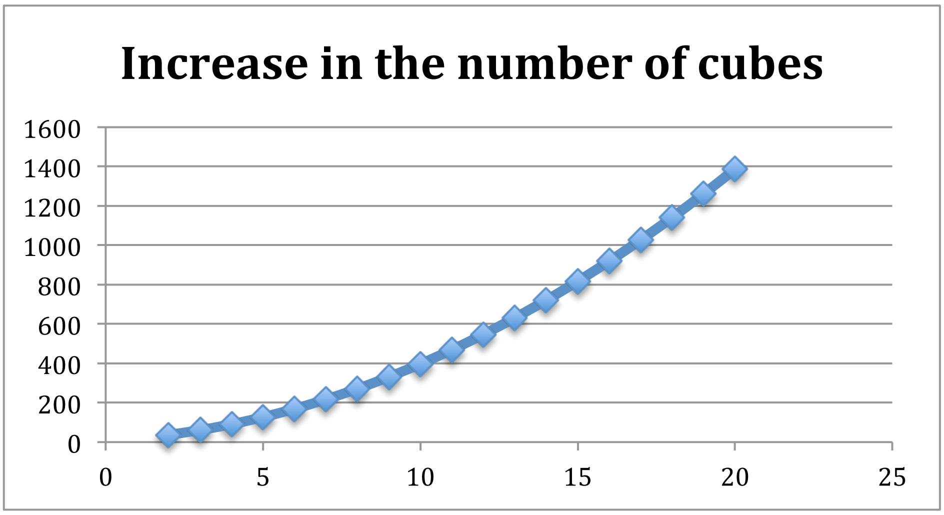
$6n^2 + 24n - 12n + n^3$

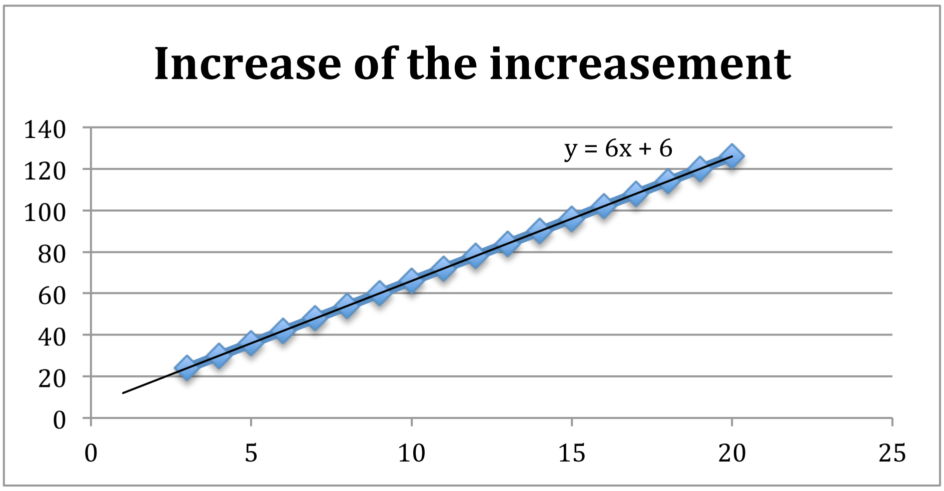
$\mathbf{V = n^3 + 6n^2 + 12n}$

Solution f)

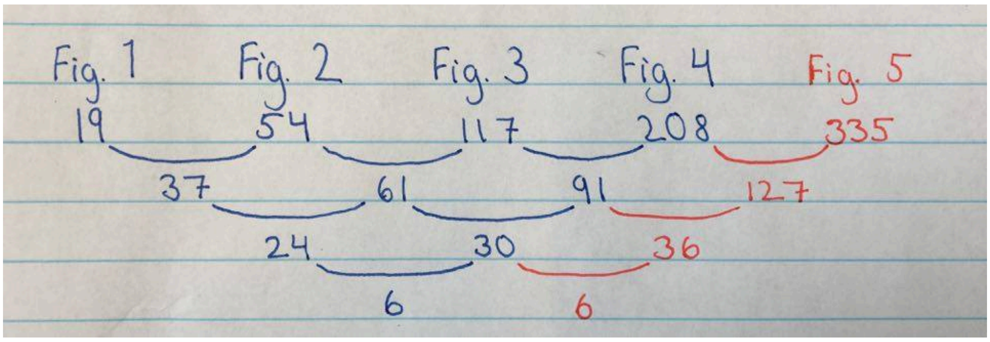
Increasement of cubes in the figures.







In the first graph, the points show the number of cubes in every figure. The x-axis shows the figure number. That means that the graph shows us the increase in the number of cubes.
In the second graph, the points show the increase in the number of cubes. That means that the graph shows the increase of the increasement of the number of cubes.
In the third graph, the points show the increase of the increasement, which means that the graph shows the increase of the increasement's increase. We can see that the increase of the increasement's increase is equal to 6 ($y = 6x + 6$).
If we know the number of cubes in figure number 1, 2, 3 and 4, we can find the number of cubes in figure 5. First we find the increase in the number of cubes from fig. 1 to fig. 2, from fig. 2 to fig. 3, and then from fig. 3 to fig. 4. Thereafter we find the increase in the increasements. The increase in the increasement's increase is 6. Have a look at the illustration.



We are using the knowledge from before to create this 'stair.' Look at the red. We add 6 to the increase of the increasement (in this case 30). Thereafter we add the sum to the increase (in this case 91). Then we add our new sum to the number of cubes in the former figure, and we get the number of cubes in next figure.

This, however, is not the smartest solution. It is complicated and takes a lot of time and later in the report, when we evaluate the solutions, this will not be included.

Comparison of solutions:

When we reduce our equations, they all end up being the same: $V = n^3 + 6n^2 + 12n$. So our different solutions are just various ways of getting the same result. We tested the equations and with both the example figure number = 4 and the example figure number = 5, the equations add up. It is faster to use equations, but easier to understand if you built the figures, though you get the same result.
Solution d) and f) stand out by not being based on a division of the figures, while solution b), c) and e) are based on dividing the figures in minor parts.
We also concluded that the solutions b), c), d) and e) can be reduced to the same equation.

b)

Evaluation of solutions:

The easy solution:

We think that the equation $\mathbf{V = (n + 2)^3 – 8}$ is an easy solution. It is not hard to use and it is easy to keep track of the numbers. It is not hard to explain how it works, it is easy to understand and manageable. We find this solution both easy and smart.

Another sense of easiness is a solution that does not take a lot of thinking. It is not difficult to build the figure and count how many centicubes you use which makes this solution the easiest to understand.

The complicated solution

We find the solution $\mathbf{V = (n^2 + n * 4) * 6 – 12n + n^3}$ complicated because there is a lot of numbers to keep track of. This makes it difficult to manage. Furthermore it is hard to understand, especially if you do not know the idea behind it.

The smart solution

All the equations can be reduced to the equation: $\mathbf{V = n^3 + 6n^2 + 12n}$**.** Accordingly, we think that this solution is the best. It is simple and easy to understand and it does not have too many numbers to keep track of.

c)

Visible surfaces on the cubes

0 visible surfaces

All the figures have a core consisting of x number of cubes with 0 visible surfaces. If you remove the outer layer you will be able to calculate that the number of cubes with 0 visible surfaces is $\mathbf{n^2}$**.**

1 visible surface

On every side of the cube there is n^2 cubes with 1 visible surface. The figure has 6 sides.

Therefore $\mathbf{n^2 * 6}$ = the number of cubes with 1 visible surface**.**

2 visible surfaces

On figure number 4 there are 2 cubes per "bar" with 2 visible surfaces. We see that on the bars the number of cubes with 2 visible surfaces is 2 smaller than the figure number.

Therefore **(n – 2) * 12 =** the number of cubes with 2 visible surfaces**.** This does not count for figure number 1, which in general is an exception. Figure number 1 has no cubes with 2 visible surfaces.

3 visible surfaces

The only cubes with 3 visible surfaces are the 3 cubes that surround every corner. All the figures have 8 corners and therefore the number of cubes with 3 visible surfaces = **3 * 8.** Figure number 1 also stands out here because it doesn't have any cubes with 2 visible surfaces.

4 visible surfaces

The only figure that has cubes with 4 visible surfaces is figure number 1. We counted that it has 12 cubes with 4 visible surfaces.

d)

Connection between answers in a) and c)

You can use the solutions in c) to create equations in a). For instance, we did that with the equation $V = (n^2 + n * 4) * 6 – 12n + n^3$. This equation is divided in cubes *without* any visible surfaces and cubes *with* visible surfaces. (See former description of the equation, **Solution e)**)

You can add the number of cubes in a figure with respectively 0, 1, 2, 3 and 4 visible surfaces, and that gives you the total number of cubes in the figure.

Example: Figure 5.

The total number of cubes is equal to $(n + 2)^3 – 8$.

There are 5^3 cubes with 0 visible surfaces, $6 * 5^2$ cubes with 1 visible surface, $(5 – 2) * 12$ with 2 visible surfaces and $3 * 8$ with 3 visible surfaces.

$5^3 + 6 * 5^2 + (5 – 2) * 12 + 3 * 8 = (5 + 2)^3 – 8$

$125 + 6 * 16 + 3 * 12 + 24 = 7^3 – 8$

$150 + 36 + 149 = 343 – 8$

$335 = 335$

The equation $n^3 + 6n^2 + 12n$ is constructed the same way: n^3 is the cubes with 0 visible surfaces, $6n^2$ is the cubes with 1 visible surface and $12n$ is the cubes with 2, 3 or 4 visible surfaces.

Conclusion

We believe that we worked concentrated to solve the assignment and that we reached a satisfying result. All of us feel like we learned a lot from this assignment. For instance, we learned to use many different working methods. We think that we answered all the tasks but that we could have been a little more creative with our solutions. We think that the best solution is the equation $\mathbf{V = (n+2)^3 - 8}$. It is easy to use and understand and not hard to keep track of the numbers. We believe that the equation $\mathbf{V = n^3 + 6n^2 + 12n}$ also makes sense because it, too, is easy to use and understand and that this equation maybe is the best fitting for the figures.